CONDENSED LESSON

# 5.1 Solving Systems of Equations

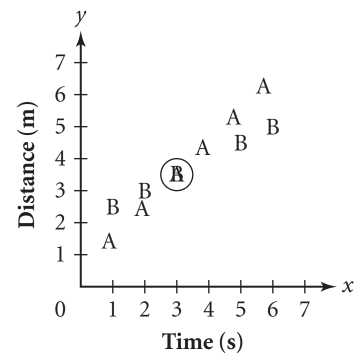
In this lesson you will

- represent situations with **systems of equations**
- use tables and graphs to **solve systems of linear equations**

A **system of equations** is a set of two or more equations with the same variables. A solution of a system of equations is a set of values that makes all the equations true.

## Investigation: Where Will They Meet?

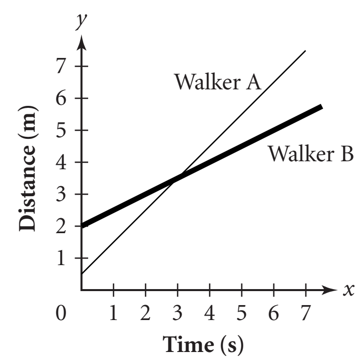
**Steps 1–4** In this investigation, two students walk along a 6-meter segment. Walker A starts at the 0.5-meter mark and walks toward the 6-meter mark at a rate of 1 m/s. Walker B starts at the 2-meter mark and walks toward the 6-meter mark at a rate of 0.5 m/s. Here is a graph of the data collected by one group.




**Steps 5–8** You can model this situation with a system of equations and then solve the system to figure out when and where Walker A passes Walker B. If $x$ represents the time in seconds and $y$ represents the distance from the 0-meter mark, the system is

$$\begin{cases} y = 0.5 + x & \text{Walker A} \\ y = 2 + 0.5x & \text{Walker B} \end{cases}$$

Here are graphs of the equations on the same axes. The graphs appear to intersect at (3, 3.5), indicating that Walker A passes Walker B after 3 seconds, when both walkers are at the 3.5-meter mark.



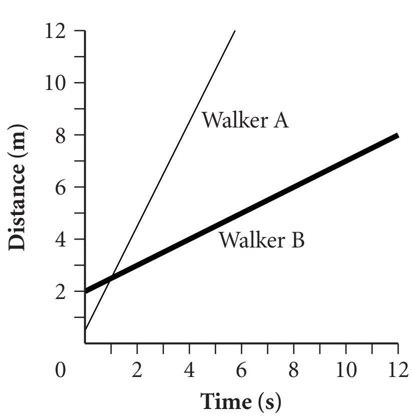

To check that (3, 3.5) really is the point of intersection, substitute 3 for $x$ and 3.5 for $y$ in both equations, and check that true statements result.

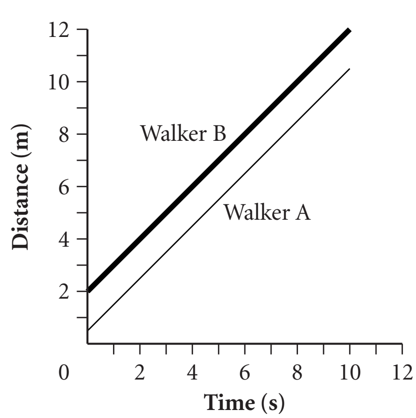
$3.5 \stackrel{?}{=} 0.5 + 3$ $\qquad$ $3.5 \stackrel{?}{=} 2 + 0.5(3)$

$3.5 = 3.5$ $\qquad$ $3.5 = 3.5$

**Steps 9–11** If Walker A moved faster than 1 m/s, the slope of Walker A's line would increase and the intersection point would move closer to the origin, indicating that Walker A passes Walker B sooner and closer to the 0-meter mark.

If the two walkers moved at the same speed, they would never meet. The slopes of the lines would be equal, so the lines would be parallel. The system of equations for this situation has no solution.




(continued)

If both walkers walked at the same speed from the same starting mark, the two lines would be identical. Every point on the line is a solution of the system, indicating that the walkers are always at the same location at the same time.

The investigation shows that two lines can intersect at zero points, at one point, or at every point. So a system of linear equations can have zero, one, or an infinite number of solutions.

Read the example in your book and then read the example below.

***EXAMPLE***

The Anytime long-distance plan charges $4.80 per month plus 5¢ a minute. The TalkMore plan charges 9¢ a minute and no monthly fee. For what number of minutes are the charges for the two plans the same?

**a.** Write a system of two equations to model this situation.

**b.** Solve the system by creating a table. Explain the real-world meaning of the solution, and locate the solution on a graph.

▶ ***Solution***

**a.** Let $x$ represent the number of minutes, and let $y$ represent the charge in dollars. The charge is the monthly fee plus the rate times the number of minutes. Here is the system of equations.

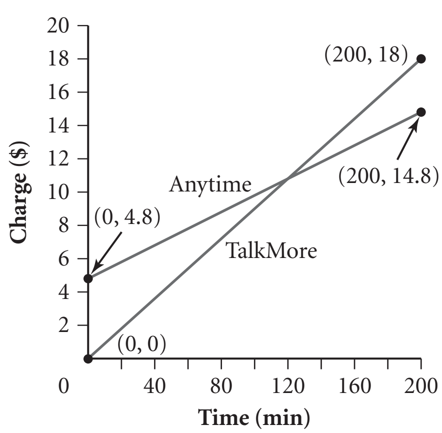
$$\begin{cases} y = 4.80 + 0.05x & \text{Anytime plan} \\ y = 0.09x & \text{TalkMore plan} \end{cases}$$

**b.** Create a table from the equations. Fill in the times and calculate the charge for each plan. The table shows that when $x = 120$, both $y$-values are 10.80. Because (120, 10.80) satisfies both equations, it is the solution of the system. The solution means that both plans charge $10.80 for 120 minutes of long-distance calls.

**Long-Distance Plans**

| Time (min) | Anytime $y = 4.80 + 0.05x$ | TalkMore $y = 0.09x$ |
|---|---|---|
| 0 | 4.80 | 0 |
| 30 | 6.30 | 2.70 |
| 60 | 7.80 | 5.40 |
| 90 | 9.30 | 8.10 |
| 120 | 10.80 | 10.80 |
| 150 | 12.30 | 13.50 |

On the graph, the solution is the point where the two lines intersect.



CONDENSED LESSON

# 5.2 Solving Systems of Equations Using Substitution

In this lesson you will

- represent situations with **systems of equations**
- use the **substitution method** to solve systems of linear equations

When you use a graph or a table to solve a system of equations, you may be able to find only an approximate solution. The s**ubstitution method** allows you to find an exact solution of a system. Read Example A in your book, which shows how to solve a system using the substitution method.

### Investigation: All Tied Up

Start with a thin rope and a thick rope, each 1 meter long. If you tie knots in each rope, measuring the length after each knot, you might get data like this.

**Thin Rope**

| Number of knots | Length (cm) |
|---|---|
| 0 | 100 |
| 1 | 94 |
| 2 | 88 |
| 3 | 81.3 |
| 4 | 75.7 |
| 5 | 69.9 |
| 6 | 63.5 |

**Thick Rope**

| Number of knots | Length (cm) |
|---|---|
| 0 | 100 |
| 1 | 89.7 |
| 2 | 78.7 |
| 3 | 68.6 |
| 4 | 57.4 |
| 5 | 47.8 |
| 6 | 38.1 |

Use the techniques you learned in Chapter 4 to write a linear equation to model the data for each rope.

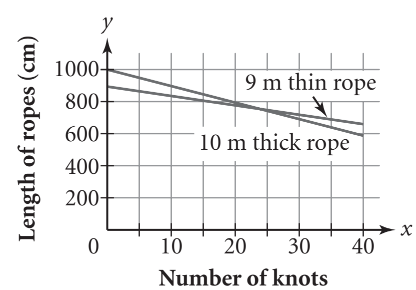
A possible model for the thin rope is $y = 100 - 6x$, where $x$ is the number of knots and $y$ is the length in centimeters. The $y$-intercept, 100, is the length of the rope before you tie any knots. The slope, $-6$, is the change in the length for each knot.

A possible model for the thick rope is $y = 100 - 10.3x$. This equation indicates that the initial length is 100 cm and that the length decreases by 10.3 cm for each knot.

Now, suppose the initial length of the thin rope is 9 meters and the initial length of the thick rope is 10 meters. This system of equations models this situation.

$$\begin{cases} y = 900 - 6x & \text{Length of thin rope} \\ y = 1000 - 10.3x & \text{Length of thick rope} \end{cases}$$

To estimate the solution of this system, make a graph and estimate the point of intersection. The intersection point is about (23, 760).




You can also find the solution by using the substitution method. Substitute $900 - 6x$ (from the first equation) for $y$ in the second equation, and solve the resulting equation.

| | |
|---|---|
| $y = 1000 - 10.3x$ | Original second equation. |
| $900 - 6x = 1000 - 10.3x$ | Substitute $900 - 6x$ for $y$. |
| $900 = 1000 - 4.3x$ | Add $6x$ to both sides and simplify. |
| $-100 = -4.3x$ | Subtract 1000 from both sides. |
| $23.26 \approx x$ | Divide both sides by $-4.3$. |

**(continued)**

Because $x$ represents the number of knots, the solution must be a whole number. So round $x$ to 23. When $x$ is 23, $y$ is about 760. So the solution is (23, 760). This means that when 23 knots have been tied in each rope, the ropes are about the same length, 760 cm.

Think about how the models would be different if the two ropes had the same thickness. In this situation the slopes would be the same, so the lines would be parallel. Thus, the system would have no solutions. In other words, the ropes would never be the same length.

If the ropes had the same thickness *and* the same starting length, the equations and the lines would be exactly the same. In this case there are many solutions. The ropes would be the same length after any number of knots had been tied.

When you solve a system using the substitution method, you sometimes need to rewrite one of the equations before you can substitute. Example B in your book shows you how to write a system of equations for a **mixture problem,** and how to solve a system when both equations are given in standard form. Read this example and the text that follows carefully. Then, read the example below.

***EXAMPLE***

Jenny is making a dried fruit mix from dried pineapple and dried mango. Dried pineapple costs \$3.95 per pound, and dried mango costs \$6.95 per pound. How much of each fruit should Jenny combine to get 3 pounds of a mixture that costs \$5 per pound?

***▶ Solution***

Let $p$ represent the number of pounds of pineapple and let $m$ represent the number of pounds of mango. This system of equations describes the situation.

$$\begin{cases} p + m = 3 \\ 3.95p + 6.95m = 3(5) \end{cases}$$

Solve one of the equations for one of the variables. For example, you might solve the first equation for $p$.

$p + m = 3$ First original equation.

$p = 3 - m$ Subtract $m$ from both sides.

Now substitute $3 - m$ for $p$ in the second original equation.

$3.95(3 - m) + 6.95m = 3(5)$ Substitute $3 - m$ for $p$.

$11.85 - 3.95m + 6.95m = 15$ Distribute and multiply.

$11.85 + 3m = 15$ Combine like terms.

$3m = 3.15$ Subtract 11.85 from both sides.

$m = 1.05$ Divide both sides by 3.

To find the corresponding $p$-value, substitute 1.05 for $m$ into one of the equations.

$p = 3 - 1.05 = 1.95$

So Jenny should buy 1.95 pounds of pineapple and 1.05 pounds of mango.

CONDENSED LESSON 5.3

# Solving Systems of Equations Using Elimination

In this lesson you will

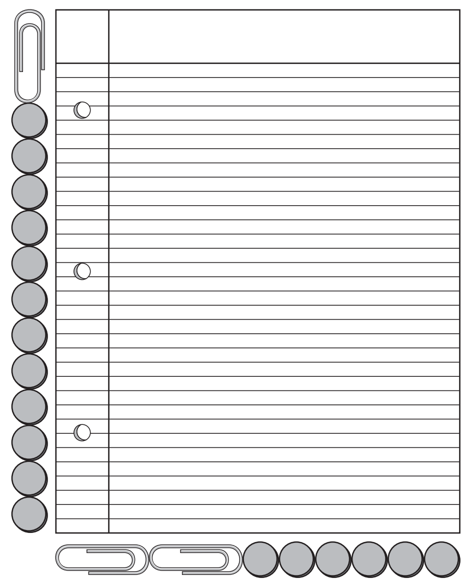
- represent situations with **systems of equations**
- use the **elimination method** to solve systems of linear equations

Read the text at the beginning of Lesson 5.3 in your book. It explains that you can add two equations to get another true equation. Then, read Example A carefully and make sure you understand it. In the example, the variable *s* is eliminated just by adding the equations. As you will see in the investigation, sometimes using the **elimination method** requires a bit more work.

### Investigation: Paper Clips and Pennies

Place one paper clip along the long side of a piece of paper. Then, line up enough pennies to complete the 11-inch length. If you use a jumbo paper clip, you should find that you need 12 pennies.

Place two paper clips along the short side of the sheet of paper, and add pennies to complete the 8.5-inch length. With jumbo paper clips, you'll need 6 pennies.



If $C$ is the length of a paper clip and $P$ is the diameter of a penny, you can write this system of equations to represent this situation.

$$\begin{cases} C + 12P = 11 & \text{Long side} \\ 2C + 6P = 8.5 & \text{Short side} \end{cases}$$

Notice that you can't eliminate a variable by adding the two original equations. However, look what happens when you multiply both sides of the first equation by $-2$.

$$\begin{cases} C + 12P = 11 \\ 2C + 6P = 8.5 \end{cases} \rightarrow \begin{cases} -2C - 24P = -22 \\ 2C + 6P = 8.5 \end{cases}$$

Because you multiplied both sides of the first equation by the same number, the new equation has the same solutions as the original. You can now eliminate the variable $C$ by adding the two equations in the new system.

$$\begin{aligned} -2C - 24P &= -22 \\ 2C + 6P &= 8.5 \\ \hline -18P &= -13.5 && \text{Add the equations.} \\ P &= 0.75 && \text{Divide by } -18. \end{aligned}$$

To find the value of $C$, substitute 0.75 for $P$ in either equation and solve for $C$.

$C + 12(0.75) = 11$ or $2C + 6(0.75) = 8.5$

**(continued)**

You should find that $C$ is 2. Be sure to check the solution by substituting 0.75 for $P$ and 2 for $C$ in both equations.

$2 + 12(0.75) = 11$ and $2(2) + 6(0.75) = 8.5$

The solution $(P, C) = (0.75, 2)$ means that the penny has a diameter of 0.75 inch and the paper clip has a length of 2 inches.

There are several ways you could have solved the original system of equations. For example, instead of multiplying the first equation by $-2$, you could have multiplied the second equation by $-2$. Then, the coefficient of $P$ would be 12 in one equation and $-12$ in the other, and you could eliminate $P$ by adding the equations.

Read the rest of the lesson in your book. Here is an additional example.

***EXAMPLE*** At Marli's Discount Music Mart, all CDs are the same price and all cassette tapes are the same price. Rashid bought six CDs and five cassette tapes for \$117.78. Quincy bought four CDs and nine cassette tapes for \$123.74. Write and solve a system of equations to find the price of a CD and the price of a cassette tape.

▶ ***Solution*** If $c$ is the price of a CD and $t$ is the price of a tape, then the problem can be modeled with this system.

$$\begin{cases} 6c + 5t = 117.78 & \text{Rashid's purchase} \\ 4c + 9t = 123.74 & \text{Quincy's purchase} \end{cases}$$

If you multiply the first equation by 2 and the second equation by $-3$, you will be able to add the equations to eliminate $c$.

$$6c + 5t = 117.78 \rightarrow \quad 12c + 10t = 235.56 \qquad \text{Multiply both sides by 2.}$$

$$4c + 9t = 123.74 \rightarrow \underline{-12c - 27t = -371.22} \qquad \text{Multiply both sides by } -3.$$

$$-17t = -135.66 \qquad \text{Add the equations.}$$

$$t = 7.98 \qquad \text{Divide.}$$

To find the value of $c$, substitute 7.98 for $t$ in either original equation and solve for $t$.

$$6c + 5t = 117.78 \qquad \text{Original first equation.}$$

$$6c + 5(7.98) = 117.78 \qquad \text{Substitute 7.98 for } t.$$

$$6c + 39.90 = 117.78 \qquad \text{Multiply.}$$

$$6c = 77.88 \qquad \text{Subtract 39.90 from both sides.}$$

$$c = 12.98 \qquad \text{Divide both sides by 6.}$$

Cassette tapes cost \$7.98 and CDs cost \$12.98. Be sure to check this solution by substituting it into both original equations.

CONDENSED LESSON 5.4

# Solving Systems of Equations Using Matrices

In this lesson you will

- represent situations with **systems of equations**
- use **matrices** to solve systems of linear equations

You now know how to solve systems of equations with tables and graphs and by using the substitution and elimination methods. You can also solve systems of equations by using matrices. Pages 296 and 297 of your book explain how to represent a system of equations with a matrix and then use row operations to find the solution. Read this text and Example A carefully.

## Investigation: Diagonalization

Consider this system of equations.

$$\begin{cases} 2x + y = 11 \\ 6x - 5y = 9 \end{cases}$$

Because the equations are in standard form, you can represent the system with a matrix. Write the numerals from the first equation in the first row, and write the numerals from the second equation in the second row.

$$\begin{bmatrix} 2 & 1 & 11 \\ 6 & -5 & 9 \end{bmatrix}$$

To solve the equation, perform row operations to get 1's in the diagonal of the matrix and 0's above and below the diagonal as shown here.

$$\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \end{bmatrix}$$

To get a 0 as the first entry in the second row, add $-3$ times the first row to the second row. This step is similar to using the elimination method to eliminate $x$ from the second equation.

| | | | | | |
|---|---|---|---|---|---|
| $-3$ times row 1 $\rightarrow$ | | $-6$ | $-3$ | $-33$ | |
| $+$ row 2 $\rightarrow$ | $+$ | $6$ | $-5$ | $9$ | |
| New row 2 $\rightarrow$ | | $0$ | $-8$ | $-24$ | |

**New matrix**

$$\begin{bmatrix} 2 & 1 & 11 \\ 0 & -8 & -24 \end{bmatrix}$$

To get 1 as the second entry in the second row, divide that row by $-8$.

$$\begin{bmatrix} 2 & 1 & 11 \\ 0 & 1 & 3 \end{bmatrix}$$

From the second row, you can see that $y = 3$. Now, subtract the second row from the first to get a 0 as the second entry in the first row. This is similar to substituting 3 for $y$ in the first equation to get $2x = 8$.

| | | | | |
|---|---|---|---|---|
| Row 1 $\rightarrow$ | | $2$ | $1$ | $11$ |
| $-$ row 2 $\rightarrow$ | $-$ | $0$ | $1$ | $3$ |
| New row 1 $\rightarrow$ | | $2$ | $0$ | $8$ |

**New matrix**

$$\begin{bmatrix} 2 & 0 & 8 \\ 0 & 1 & 3 \end{bmatrix}$$

**(continued)**

To get a 1 as the first entry in the first row, divide the row by 2.

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 3 \end{bmatrix}$$

You can now see that $x = 4$ and $y = 3$. You can check this solution by substituting it into the original equation.

Example B in your book shows that matrices are useful for solving systems of equations involving large numbers. Here is another example.

***EXAMPLE***

At a college football game, students paid \$12 per ticket and nonstudents paid \$18 per ticket. The number of students who attended was 1,430 more than the number of nonstudents. The total of all ticket sales was \$67,260. How many of the attendees were students, and how many were nonstudents?

▶ ***Solution***

If $S$ is the number of students and $N$ is the number of nonstudents, then you can represent the situation with this system and matrix.

$$\begin{cases} S - N = 1{,}430 \\ 12S + 18N = 67{,}260 \end{cases} \rightarrow \begin{bmatrix} 1 & -1 & 1{,}430 \\ 12 & 18 & 67{,}260 \end{bmatrix}$$

Use row operations to find the solution.

Add $-12$ times row 1 to row 2 to get new row 2. $\begin{bmatrix} 1 & -1 & 1{,}430 \\ 0 & 30 & 50{,}100 \end{bmatrix}$

Divide row 2 by 30. $\begin{bmatrix} 1 & -1 & 1{,}430 \\ 0 & 1 & 1{,}670 \end{bmatrix}$

Add row 2 to row 1 to get new row 1. $\begin{bmatrix} 1 & 0 & 3{,}100 \\ 0 & 1 & 1{,}670 \end{bmatrix}$

The final matrix shows that $S = 3{,}100$ and $N = 1{,}670$. So 3,100 students and 1,670 nonstudents attended the game. You can check this solution by substituting it into both original equations.

CONDENSED LESSON

# 5.5 Inequalities in One Variable

In this lesson you will

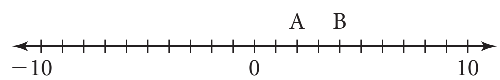
- write **inequalities** to represent situations
- learn how applying operations to both sides of an inequality affects the direction of the inequality symbol
- solve a problem by writing and **solving an inequality**

An **inequality** is a statement that one quantity is less than or greater than another. Inequalities are written using the symbols $<$, $>$, $\leq$, and $\geq$. Read the text on page 304 of your book, which gives several examples from everyday life and how to write them as inequalities.

Just as with equations, you can solve inequalities by applying the same operations to both sides. However, as you will learn in the investigation, you need to be careful about the direction of the inequality symbol.

## Investigation: Toe the Line

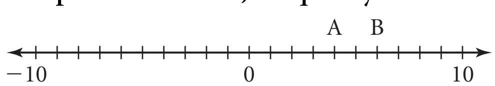
In this investigation, two walkers stand on a number line. Walker A starts on the number 2, and Walker B starts on the number 4. You can represent this situation with the inequality $2 < 4$.



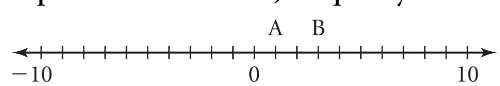

**Steps 1–4** When an announcer calls out an operation, the walkers perform the operation on their numbers and move to new positions based on the result. The new positions are represented by an inequality, with the position of Walker A on the left side and the position of Walker B on the right side.

The drawings below show the walkers' positions after the first two operations along with the corresponding inequality.

**Operation: Add 2; Inequality: $4 < 6$**




**Operation: Subtract 3; Inequality: $1 < 3$**




This table shows the results of the remaining operations.

| Operation | Walker A's position | Inequality symbol | Walker B's position |
|---|---|---|---|
| Add $-2$ | $-1$ | $<$ | 1 |
| Subtract $-4$ | 3 | $<$ | 5 |
| Multiply by 2 | 6 | $<$ | 10 |
| Subtract 7 | $-1$ | $<$ | 3 |
| Multiply by $-3$ | 3 | $>$ | $-9$ |
| Add 5 | 8 | $>$ | $-4$ |
| Divide by $-4$ | $-2$ | $<$ | 1 |
| Subtract 2 | $-4$ | $<$ | $-1$ |
| Multiply by $-1$ | 4 | $>$ | 1 |

(continued)

**Steps 5–9** Notice that when a number is added to or subtracted from the walkers' positions, the direction of the inequality (that is, the relative positions of the walkers) remains the same. The direction of an inequality also stays the same when the positions are multiplied or divided by a positive number. However, when the positions are multiplied or divided by a negative number, the direction of the inequality (that is, the relative positions of the walkers) is reversed.

Check these findings by starting with another inequality and applying operations to both sides. You should find that *the direction of the inequality symbol is reversed only when you multiply or divide by a negative number.*

Read Example A in your book, which shows how to graph solutions to inequalities on a number line. Then, read Example B, which applies what you learned in the investigation to solve an inequality. Here is an additional example.

***EXAMPLE A*** Jack takes the bus to the bowling alley. He has \$15 when he arrives. It costs \$2.25 to bowl one game. If Jack needs \$1.50 to take the bus home, how many games can he bowl? Solve this problem by writing and solving an inequality.

▶ ***Solution*** Let $g$ represent the number of games Jack can bowl. We know that the amount Jack starts with minus the amount he spends bowling must be at least (that is, greater than or equal to) \$1.50. So we can write this inequality.

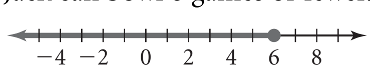
Amount Jack starts with → 15; Cost of bowling $g$ games → $2.25g$; Bus fare → 1.50

$$15 - 2.25g \geq 1.50$$

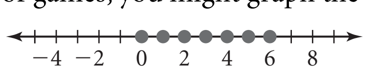
Now, solve the inequality.

| | |
|---|---|
| $15 - 2.25g \geq 1.50$ | Original inequality. |
| $15 - 15 - 2.25g \geq 1.50 - 15$ | Subtract 15 from both sides. |
| $-2.25g \geq -13.50$ | Subtract. |
| $\dfrac{-2.25g}{-2.25} \leq \dfrac{-13.50}{-2.25}$ | Divide both sides by $-2.25$, and *reverse the inequality symbol.* |
| $g \leq 6$ | Divide. |

Jack can bowl 6 games or fewer. Here, $g \leq 6$ is graphed on a number line.




Because Jack can bowl no fewer than 0 games, and he can bowl only a whole number of games, you might graph the solution like this:



CONDENSED LESSON **5.6**

# Graphing Inequalities in Two Variables

In this lesson you will

- **graph linear inequalities** in two variables

You know how to graph linear equations in two variables, such as $y = 6 - 3x$. In this lesson you will learn to graph linear inequalities in two variables, such as $y < 6 - 3x$ and $y \geq 6 - 3x$.

## Investigation: Graphing Inequalities

To complete this investigation, you'll need a grid worksheet like the one on page 312 of your book.

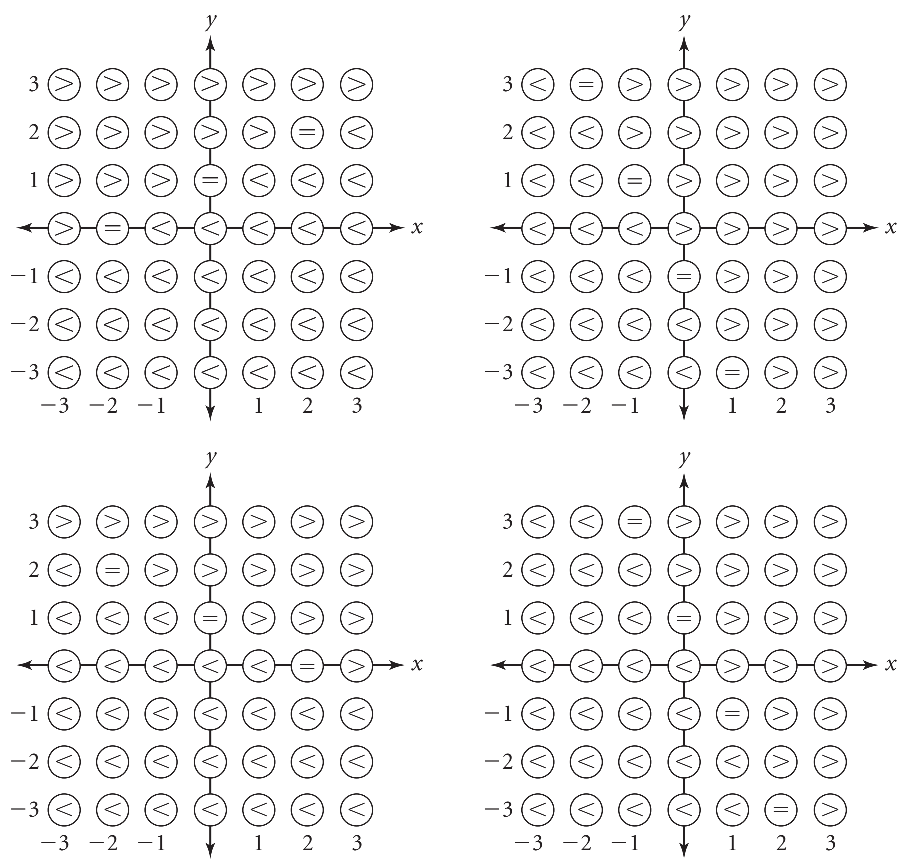
Choose one of the statements listed on page 312. For each point shown with a circle on the worksheet, substitute the coordinates of the point into the statement, and then fill in the circle with the relational symbol, $<$, $>$, or $=$, that makes the statement true. For example, if you choose the statement $y \square -1 - 2x$, do the following for the point (3, 2):

$y \square -1 - 2x$ Original statement.

$2 \square -1 - 2(3)$ Substitute 3 for $x$ and 2 for $y$.

$2 \square -7$ Subtract.

Because the symbol $>$ makes this statement true, write $>$ in the circle corresponding to the point (3, 2). Here are completed grids for the four statements.



**(continued)**

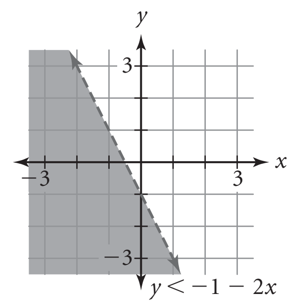
Notice that for each statement, the circles containing equal signs form a straight line. All the circles above the line are filled in with $>$ symbols, and all the circles below the line are filled in with $<$ symbols.

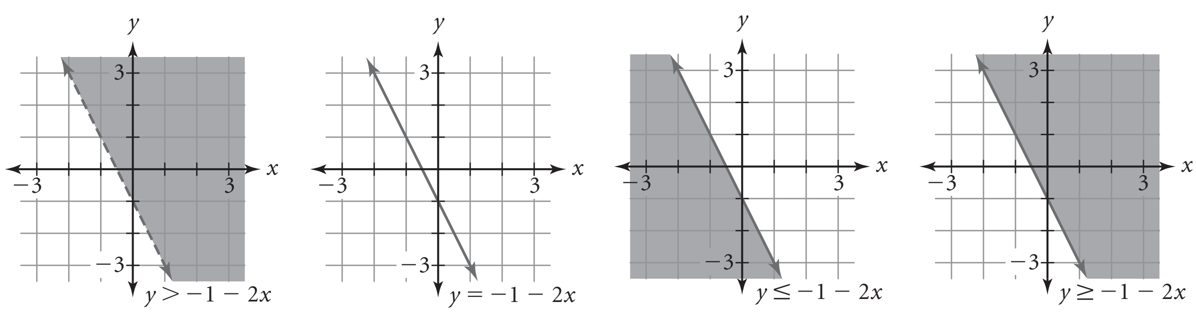
Choose one of the statements and test a point with fractional or decimal coordinates. For example, in the grid for $y \square -1 - 2x$, $(-2.2, 1.5)$ is below the line of equal signs. Substitute the coordinates into the statement.

| | |
|---|---|
| $y \square -1 - 2x$ | Original statement. |
| $1.5 \square -1 - 2(-2.2)$ | Substitute $-2.2$ for $x$ and 1.5 for $y$. |
| $1.5 \square 3.4$ | Subtract. |
| $1.5 < 3.4$ | Insert the appropriate symbol. |

The resulting statement gets a $<$ symbol, just like the other points below the line of equal signs.

Shown here are graphs of $y < -1 - 2x$, $y > -1 - 2x$, $y = -1 - 2x$, $y \leq -1 - 2x$, and $y \geq -1 - 2x$. In each graph the shaded regions include the points that make the statement true. A dashed line indicates that the line is *not* included in the graph. A solid line indicates that the line is included.




Make similar graphs for the other inequalities. You should notice the following:

- Graphs of inequalities in the form $y >$ *expression* and $y \geq$ *expression* are shaded above the line.
- Graphs of inequalities in the form $y <$ *expression* and $y \leq$ *expression* are shaded below the line.
- Graphs of inequalities in the form $y \leq$ *expression* and $y \geq$ *expression* require a solid line.
- Graphs of inequalities in the form $y <$ *expression* and $y >$ *expression* require a dashed line.

Read the rest of the lesson and the example in your book. When you are finished, you should be able to graph any linear inequality.

CONDENSED LESSON

# 5.7 Systems of Inequalities

In this lesson you will

- **graph solutions** of systems of inequalities
- use systems of inequalities to represent situations involving **constraints**

You can find the solution to a system of equations by graphing the equations and locating the points of intersection. You can use a similar method to find the solution to a system of inequalities. When a real-world problem is represented with a system of inequalities, the inequalities are often called **constraints.**

### Investigation: A "Typical" Envelope

Here are two constraints the U.S. Postal Service imposes on envelope sizes.

- The ratio of length to width must be less than or equal to 2.5.
- The ratio of length to width must be greater than or equal to 1.3.

If $l$ and $w$ represent the length and width of an envelope, then the first constraint can be represented by the equation $\frac{l}{w} \leq 2.5$, and the second can be represented by $\frac{l}{w} \geq 1.3$.

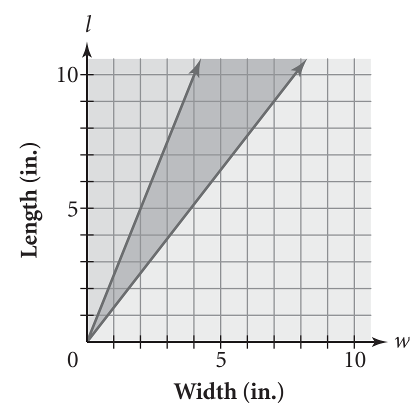
You can solve each inequality for $l$ by multiplying both sides by $w$. This gives the system

$$\begin{cases} l \leq 2.5w \\ l \geq 1.3w \end{cases}$$

Note that you do not need to reverse the direction of the inequality symbol when you multiply both sides by $w$ because the width of an envelope must be a positive number.

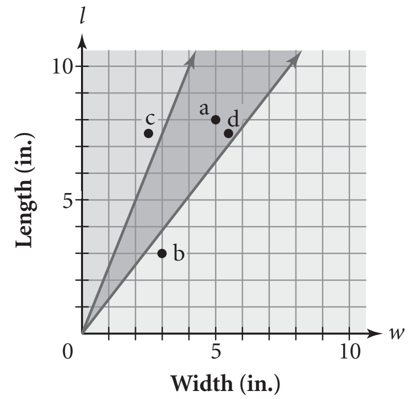
Here, both inequalities are graphed on the same axes.

The overlap of the shaded regions is the solution of the system. You can check this by choosing a point from the overlapping region and making sure its coordinates satisfy both inequalities.



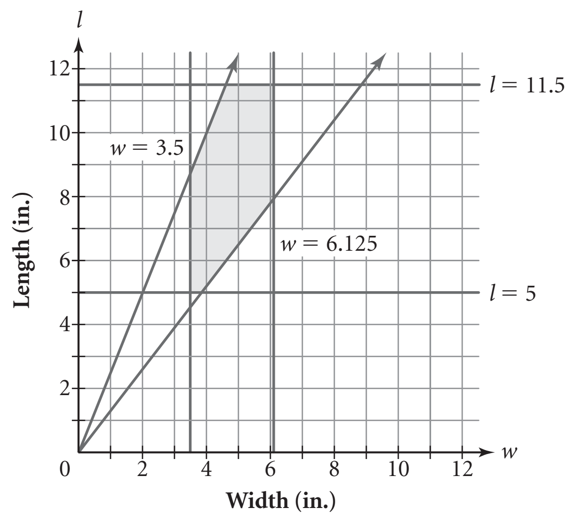

(continued)

Step 5 in your book gives the dimensions of four envelopes. Points corresponding to these envelopes are plotted on the graph here. Point *a,* which corresponds to a 5 in.-by-8 in. envelope, and Point *d,* which corresponds to a 5.5 in.-by-7.5 in. envelope, fall within the overlapping regions, indicating that these envelopes satisfy both constraints.




Notice that (0, 0) satisfies the system. This point corresponds to an envelope with no length or width, which does not make sense. Adding constraints specifying minimum and maximum lengths and widths would make the system a more realistic model. For example, for an envelope to require a 34¢ stamp, the length must be between 5 in. and 11.5 in. and the width must be between 3.5 in. and 6.125 in. The system includes these constraints and has this graph.

$$\begin{cases} l \le 2.5w \\ l \ge 1.3w \\ l \ge 5 \\ w \ge 3.5 \\ l \le 11.5 \\ w \le 6.125 \end{cases}$$




Read Examples A and B in your book. Then, read the additional example below.

***EXAMPLE*** Graph this system of inequalities and indicate the solution.

$$\begin{cases} y \ge 3 - 2x \\ y < -2 + \frac{3}{4}x \end{cases}$$

▶ ***Solution*** Graph $y = 3 - 2x$ with a solid line because its points satisfy the inequality. Shade above the line because its inequality has the "greater than or equal to" symbol.

Graph $y = -2 + \frac{3}{4}x$ with a dashed line because its points do not satisfy the inequality. Shade below the line because in the inequality, $y$ is "less than" the expression in $x$.

The points in the overlapping region satisfy both inequalities, so the overlapping region is the solution of the system.