CONDENSED LESSON

7.1 Secret Codes

In this lesson you will

- use a **coding grid** to write a coded message
- create and use a **letter-shift code**
- determine whether given relationships are **functions**

You have studied many relationships between variables. In this lesson you will learn about a special type of relationship called a *function.*

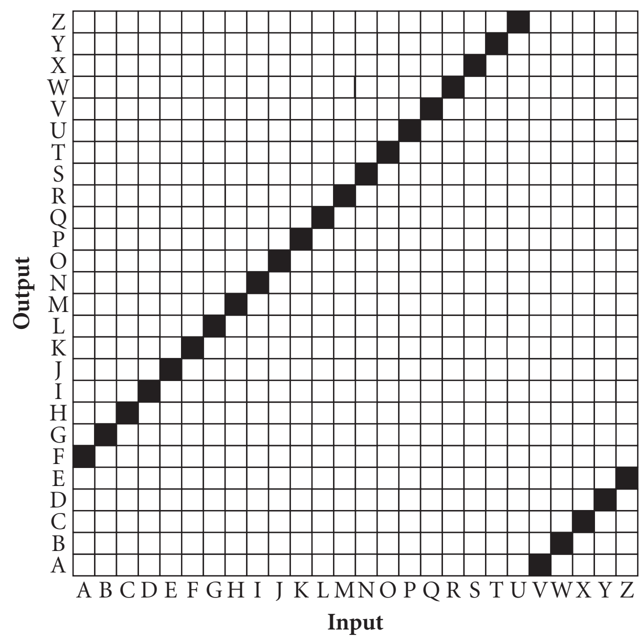
Investigation: TFDSFU DPEFT

The table and grid on pages 388 and 389 of your book represent a letter-shift code. Read the text before Step 1, which explains how to use the code to write messages.

Steps 1–3 Use the coding grid to write a short coded message. For example, JHO TUSETYDW JXYI is the code for TRY DECODING THIS.

Steps 3–6 Now, create your own code by writing a rule that shifts the letters a specified number of places. Then, put your code on a grid. The grid shows a code in which letters are shifted by 5 letters.

Using this grid, the message TRY DECODING THIS is coded YWD IJHTISL YMNX.



For any letter-shift code, the grid will show two parallel lines of shaded squares, with no row or column containing more than one shaded square.

Steps 7–11 Page 390 shows a grid for a different code. Try using the grid to decode this word: MHJLH.

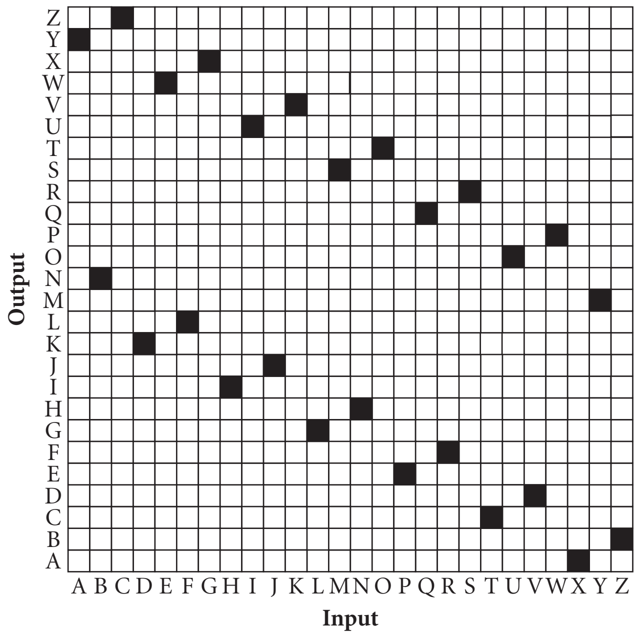
The coded word is WATER. Were you able to decode it? You may have had trouble because some of the code letters could represent two possible letters in the actual word. For example, H could represent A or R, and J could represent C or T.

Now, use the grid to code the word FUNCTION. You should find that there are several possible codes. The grid indicates that every letter between K and S can be coded in two different ways.

The grid on page 389 makes it much easier to code and decode messages because each input letter corresponds to exactly one output letter.

(continued)

Use a coding grid to create a coding scheme in which each input letter corresponds to one output letter and no two shaded squares touch one another. This grid shows one possibility.



The codes you looked at in the investigation are relationships. A relationship, such as a letter-shift code, for which there is exactly one output for each input, is called a **function.** The set of input values for a function is the **domain** of the function, and the set of output values is the **range.** In the letter-shift code, the domain and range are the same, but this is not true for all functions.

Read the text and example that follow the investigation in your book. Here is another example.

EXAMPLE

Tell whether each table represents a function.

a.

Input	Output
1	4
2	3
3	4
4	3

b.

Input	Output
red	rose
blue	sky
yellow	sun
blue	ocean

c.

Input	Output
A	a
B	b
C	c
D	d

▶ ***Solution***

a. Each input has exactly one output, so this is a function.

b. The input blue has two outputs (sky and ocean), so this is not a function.

c. Each input has exactly one output, so this is a function.

CONDENSED LESSON

7.2 Functions and Graphs

In this lesson you will

- represent relationships with tables, graphs, and equations
- use the **vertical line test** to determine whether a relationship is a function

You have written and used many rules that transform one number into another. For example, one simple rule is "Multiply each number by 2." You can represent this rule with a table, an equation, a graph, or a diagram.

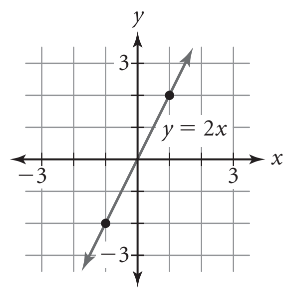
Table

Input x	Output y
3	6
-14	-28
9.3	18.6
x	$2x$

Equation

$y = 2x$

Graph



Diagram

Domain → Range

3 → 6

-14 → -28

9.3 → 18.6

In this lesson you will learn a method for determining whether a rule is a function based on its graph.

Investigation: Testing for Functions

Page 397 shows four tables, four algebraic statements (equations or inequalities), and four graphs. You'll decide whether each relationship is a function.

Step 1 Look at Table 1. Each input has only one output, so the relationship is a function. In Table 2, the input values 1 and 4 each have two different possible outputs: the x-value 1 has corresponding y-values of -1 and 1, and the x-value 4 has corresponding y-values of 2 and -2. So Table 2 does not represent a function. Table 3 represents a function and Table 4 does not. Can you see why?

Step 2 Consider Statement 1, $y = 1 + 2x$. For any x-value that you input, you multiply by 2, then add 1. There is only one possible output value that can result for any given input value. So Statement 1 represents a function. For Statement 2, can you think of two different y-values that correspond to a single x-value? If $x = 4$, y can be 2 or -2, so Statement 2 does not represent a function. Statement 3 represents a function, and Statement 4 does not. Can you see why?

Steps 3–4 You can move a vertical line, such as the edge of a ruler, from left to right on a graph to determine whether the graph represents a function. If the vertical line ever intersects the graph more than once, you know that there is an x-value that has more than one corresponding y-value, so the graph is not a function. Graph 1 represents a function because no vertical line will intersect the graph more than once. For Graph 2, however, any vertical line to the right of the y-axis will intersect the graph twice, so the graph is not a function. What about Graphs 3 and 4?

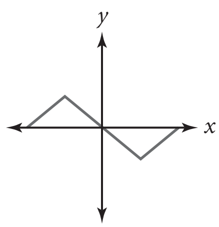
(continued)

The **vertical line test** helps you determine whether a relationship is a function by looking at its graph. If all possible vertical lines cross the graph only once or not at all, then the graph is a function. If even one vertical line crosses the graph more than once, the graph is not a function.

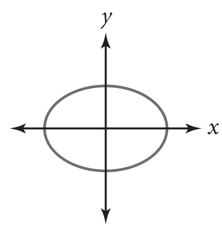
Read the rest of the lesson in your book. Then, read the example below.

EXAMPLE

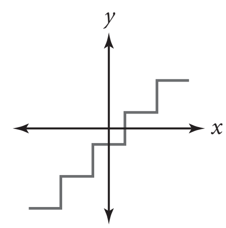
Use the vertical line test to determine which relationships are functions.



Relationship A



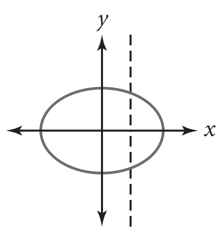
Relationship B

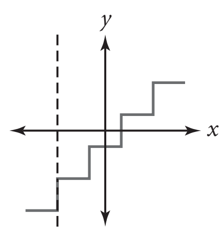


Relationship C

▶ ***Solution***

Relationship A is a function because any vertical line crosses the graph only once. Relationship B is not a function because you can draw a vertical line that crosses the graph twice. Relationship C is not a function because any vertical line through a vertical segment of the graph crosses the graph more than once.





CONDENSED LESSON

7.3 Graphs of Real-World Situations

In this lesson you will

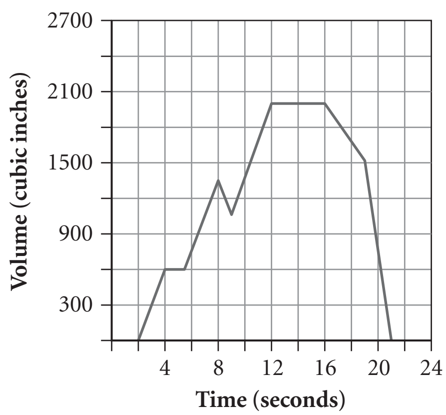
- describe graphs using the words **increasing, decreasing, linear,** and **nonlinear**
- match graphs with descriptions of real-world situations
- learn about **continuous** and **discrete** functions
- use intervals of the domain to help you describe a function's behavior

In this lesson you'll look at graphs that show how two real-world quantities are related, and you'll practice interpreting and describing graphs. Page 404 of your book discusses graphs of linear and nonlinear functions. Read this text carefully.

Example A in your book shows the relationship between time and the depth of water in a leaky swimming pool. Read the example and make sure you understand how the description of the situation fits the graph. Here is another example.

EXAMPLE

This graph shows the volume of air in a balloon as it changes over time. Tell what quantities are varying and how they are related. Give possible real-world events in your explanation.



Solution

The graph shows how the volume of air changes over time. The balloon is completely deflated for about the first 2 seconds, that is, for $0 \leq t \leq 2$. From 2 seconds to about 4 seconds ($2 \leq t \leq 4$), the balloon is inflated at a fairly steady rate. Between the 4- and 5.5-second marks, the volume stays constant at about 600 cubic inches. Perhaps the person blowing up the balloon is holding the balloon closed while she takes a breath. In the period $5.5 \leq t \leq 8$, the balloon is being inflated again. From $t = 8$ to about $t = 9$, the volume of air decreases slightly. The person might be taking another break but not holding the balloon closed tightly, allowing air to escape. During the period $9 \leq t \leq 12$, the balloon is inflated more. Then, from $t = 12$ to about $t = 16$, the volume is steady at about 2000 cubic inches. Perhaps the balloon is fully inflated so that the person stops blowing and holds the balloon closed. From about $t = 16$ to $t = 19$, the balloon is deflating slowly. The person may be holding the balloon partially closed. Between $t = 19$ and $t = 22$, the balloon is deflating quickly. The person might have let go of the balloon, allowing the air to escape rapidly.

(continued)

In the preceding example, the volume of air is a function of time. Because the volume of air *depends* on the time, it is the **dependent variable.** Time is the **independent variable.** Read the discussion of dependent and independent variables and domain and range on pages 404 and 405 of your book.

Investigation: Matching Up

Steps 1–2 All the graphs in Step 1 of the investigation show **increasing** functions, meaning that as the x-values increase, the y-values also increase. In Graph A, the function values increase at a constant rate. In Graph B, the values increase slowly at first and then more quickly. In Graph C, the function switches from one constant rate of increase to another.

The graphs in Step 2 show **decreasing** functions, meaning that as the x-values increase, the y-values decrease. In Graph D, the function values decrease at a constant rate. In Graph E, the values decrease quickly at first and then more slowly. In Graph F, the function switches from one constant rate of decrease to another.

Steps 3–5 Now, look at the graphs on page 406 of the investigation. Then, read the description of Situation A. The independent variable for this situation is time, and the dependent variable is the number of deer. Think about how the population changes over time, and make a sketch. Your sketch should look somewhat like Graph 5, which is a nonlinear graph at first and then increases at an increasing rate. Then, the rate of change slows down and the graph becomes nearly linear with a very small rate of change. This fits the description of Situation A, which states that the number of deer initially increased by a steady *percentage* (indicating exponential growth) and then the growth rate leveled off.

Read and make a sketch for Situation B. The independent variable for this case is time in days, and the dependent variable is hours of daylight. Your sketch should look somewhat like Graph 3, which is a nonlinear graph that increases slowly at first, then increases more quickly, then levels off and reaches a maximum value, then decreases quickly, and then decreases more slowly. This matches how the hours of daylight change over the course of a year.

Read and make a sketch for Situation C. The independent variable is the width of the garden, and the dependent variable is its area. Your sketch should be similar to Graph 1, which is a nonlinear graph that starts at 0, increases quickly at first, then slows down and reaches a maximum value, and then decreases, slowly at first and then more quickly. When the width is 0, so is the area. This matches the description of how the area changes with increasing width.

Read and make a sketch for Situation D. The independent variable is the time, and the dependent variable is the difference between the tea temperature and the room temperature. Your sketch should be similar to Graph 4, which is a nonlinear graph that decreases quickly at first and then more and more slowly. This matches the description of the temperature change.

Functions that have smooth graphs, with no breaks in the domain or range, are called **continuous** functions. Functions that are not continuous often involve quantities—such as people, cars, or stories of a building—that are counted or measured in whole numbers. Such functions are called **discrete** functions. Read about continuous and discrete functions on page 407 of your book.

CONDENSED LESSON

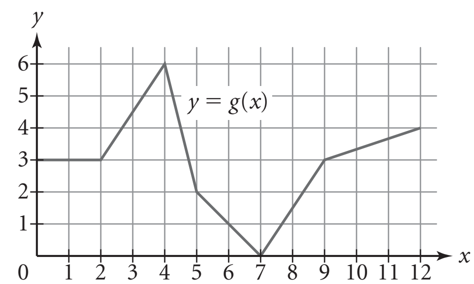
7.4 Function Notation

In this lesson you will

- learn to use **function notation**
- use a graph to evaluate a function for various input values
- use an equation to evaluate a function for various input values

The equation $y = 1 - 2x$ represents a function. You can use the letter f to name this function and then use **function notation** to express it as $f(x) = 1 - 2x$. You read $f(x)$ as "f of x," which means "the output value of the function f for the input value x." So, for example, $f(2)$ is the value of $1 - 2x$ when x is 2, so $f(2) = -3$. (Note: In function notation, the parentheses do *not* mean multiplication.)

Not all functions are expressed as equations. Here is a graph of a function g. The equation is not given, but you can still use function notation to express the outputs for various inputs. For example, $g(0) = 3$, $g(4) = 6$, and $g(6) = 1$. Can you find x-values for which $g(x) = 3$?



Investigation: A Graphic Message

Look at the graph given at the beginning of the investigation. The domain of this function is $0 \le x \le 26$, and the range is $0 \le y \le 20$.

Look at the sequence given in Step 2 in your book. Use the graph of f to evaluate the function for the given inputs, and find the value of each term of the sequence. Here are the results.

$f(3) = 5$ $\quad$ $f(18) + f(3) = 16 + 5 = 21$ $\quad$ $f(5) \cdot f(4) = 3 \cdot 4 = 12$

$f(15) \div f(6) = 10 \div 2 = 5$ $\quad$ $f(20) - f(10) = 20 - 2 = 18$

So the sequence is 5, 21, 12, 5, 18.

Now, look at the sequence given in Step 3. Evaluate f for the given inputs, and find the value of each term of the sequence. Here are the results.

$f(0) + f(1) - 3 = 8 + 7 - 3 = 12$

$5 \cdot f(9) = 5 \cdot 1 = 5$

When $f(x) = 10$, $x = 15$

$f(9 + 8) = f(17) = 14$

$\frac{f(17) + f(10)}{2} = \frac{14 + 2}{2} = 8$

$f(8 \cdot 3) - 5 \cdot f(11) = f(24) - 5 \cdot 3 = 16 - 15 = 1$

$f(4 \cdot 5 - 1) = f(19) = 18$

$f(12) = 4$

So the sequence is 12, 5, 15, 14, 8, 1, 18, 4.

(continued)

Now, think of the numbers 1 through 26 as the letters A through Z. Replace each number in the sequences you found in Steps 2 and 3 with the corresponding letter. The sequence in Step 2 gives EULER, and the sequence in Step 3 gives LEONHARD. Leonhard Euler (pronounced "oiler") invented much of the mathematical notation in use today.

EXAMPLE

You can use the function $f(x) = -19.4 + 1.28x$ to approximate the wind chill temperature $f(x)$ for a given actual temperature when the wind speed is 15 miles per hour. Both x and $f(x)$ are in degrees Fahrenheit. Find $f(x)$ for each given value of x.

a. $f(-10)$ **b.** $f(0)$ **c.** x when $f(x) = 19$ **d.** x when $f(x) = -13$

Solution

In a and b, substitute the value in parentheses for x in the function. In c and d, substitute the given value for $f(x)$.

a.
$$\begin{aligned} f(-10) &= -19.4 + 1.28(-10) \\ &= -19.4 + (-12.8) \\ &= -32.2 \end{aligned}$$

b.
$$\begin{aligned} f(0) &= -19.4 + 1.28(0) \\ &= -19.4 + 0 \\ &= -19.4 \end{aligned}$$

c.
$$\begin{aligned} 19 &= -19.4 + 1.28x \\ 38.4 &= 1.28x \\ 30 &= x \end{aligned}$$

d.
$$\begin{aligned} -13 &= -19.4 + 1.28x \\ 6.4 &= 1.28x \\ 5 &= x \end{aligned}$$

See **Calculator Note 7A** to learn how to evaluate functions on your calculator. Your calculator uses the notation $Y_1(x)$ instead of $f(x)$. The function is the equation you have entered into Y_1. When you write an equation for a function, you can use any letters you want to represent the variables and the function. For example, you might use $W(t) = -19.4 + 1.28t$ for the wind chill function discussed above.

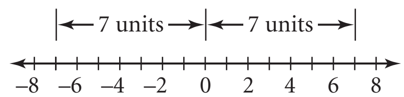
CONDENSED LESSON 7.5

Defining the Absolute-Value Function

In this lesson you will

- evaluate numerical expressions involving **absolute value**
- investigate the **absolute-value function**
- solve equations and inequalities involving the absolute value of a variable

The **absolute value** of a number is its size, or magnitude, regardless of whether the number is positive or negative. You can think of the absolute value of a number as its distance from zero on a number line. For example, both 7 and -7 are 7 units from zero, so both have an absolute value of 7.



The notation $|x|$ is used to denote the absolute value of a number or an expression. So $|7| = 7$ and $|-7| = 7$. Read Example A in your book.

Investigation: Deviations from the Mean

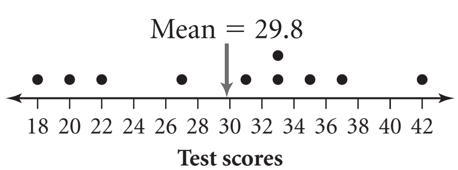
In this investigation you will learn how the absolute value is useful for describing how much a data value deviates from the mean.

Students on the math team took a difficult test to try to qualify for a statewide competition. The first column of the table at right shows their test scores. Enter the values from this column into list L1 on your calculator.

Score	Deviation (score − mean)	Distance from mean
27	−2.8	2.8
33	3.2	3.2
42	12.2	12.2
22	−7.8	7.8
37	7.2	7.2
20	−9.8	9.8
35	5.2	5.2
33	3.2	3.2
31	1.2	1.2
18	−11.8	11.8

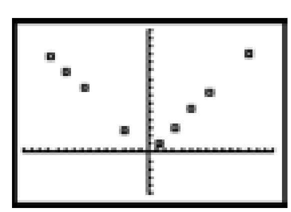
The mean of the test scores is 29.8. The second column of the table shows the difference between each score and the mean. These values show how much each score *deviates* from the mean. Enter the formula L1 $-$ 29.8 into list L2.

Here is a dot plot of the test scores.



The third column of the table shows the distance of each data value from the mean. These values are all positive. They are the *absolute values* of the deviations in the second column. Enter these values into list L3.

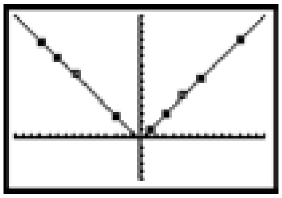
Here is a scatter plot with the L2 values (the deviations) on the x-axis and the L3 values (the distances) on the y-axis. The x-values are both positive and negative, while the y-values are all positive. If you trace the graph, you will find that positive inputs are unchanged as outputs, whereas negative inputs are changed to their opposites.



[−15, 15, 1, −5, 15, 1]

(continued)

The graph of $y = |x|$ (that is, $Y_1 = \text{abs}(x)$) has been added to the plot. Because each L_3 value is the absolute value of the corresponding L_2 value, this graph passes through all the points in the plot.



Find the mean of the deviations in list L_2 and of the distances in list L_3. The mean of the deviations is 0, and the mean of the distances is 6.44. When you are calculating the mean of the deviations, the positive and negative values "cancel each other out," resulting in a mean of 0. The mean of the distances, which are all positive, better indicates the variation in the data.

Write a rule for the absolute-value function. Here is one possible rule: If x is positive or zero, then y is equal to x. If x is negative, then y is equal to the opposite of x.

Read the rest of the lesson in your book, paying close attention to Example B, which shows how to solve an equation or inequality involving the absolute value of a variable. Here is another example.

EXAMPLE Solve each equation or inequality symbolically.

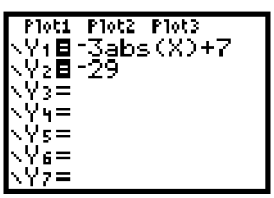
a. $-3|x| + 7 = -29$ **b.** $-3|x| + 7 > -29$

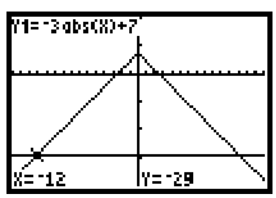
▶ ***Solution***

a.

$-3\|x\| + 7 = -29$	Original equation.
$-3\|x\| = -36$	Subtract 7 from both sides.
$\|x\| = 12$	Divide both sides by -3.
$x = 12$ or $x = -12$	Find two numbers with absolute value 12.

b. In part a you found that $-3|x| + 7 = -29$ when $x = 12$ or $x = -12$. This graph confirms those solutions.





$[-15, 15, 1, -40, 20, 10]$

You can also use the graph to help determine the solutions to the inequality $-3|x| + 7 > -29$. The graph of $Y_1 = -3|x| + 7$ is greater than the graph of $Y_2 = -29$ for x-values between -12 and 12. So the solution to $-3|x| + 7 > -29$ is $-12 < x < 12$.

CONDENSED LESSON

7.6 Squares, Squaring, and Parabolas

In this lesson you will

- calculate the **squares** of numbers
- investigate the **squaring function** and its graph
- use the **square root function** to undo the squaring function

When you multiply the number 4 by itself, you get 16. You get the same result if you multiply -4 by itself. The product of a number and itself is the **square** of the number, and the process of multiplying a number by itself is called **squaring.** The square of a number x is "x squared" and is written as x^2.

When you square numbers, you need to be careful of the order of operations. Calculate -5^2 and $(-5)^2$ on your calculator. For -5^2, your calculator squares 5 and then takes the opposite of the result, giving the output value -25. For $(-5)^2$, your calculator squares -5—that is, it multiplies -5 by itself—giving the output 25.

Investigation: Graphing a Parabola

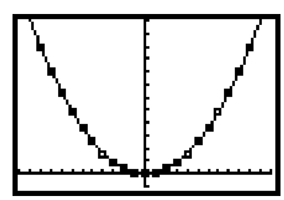
Steps 1–5 Make a two-column table with the integers -10 through 10 in the first column and the square of each integer in the second column. Square the numbers *without* using your calculator. Here is a portion of the table.

Number (x)	Square (x^2)
-4	16
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9
4	16

Now, enter the integers -10 through 10 into list L1 on your calculator. Use the x^2 key to square each number. (See **Calculator Note 7B.**) Store the result in list L2. Make sure the values in list L2 match those in your table.

Look at the values in the table. Notice that the squares of both the positive and negative numbers are positive and that the square of a number is equal to the square of its opposite.

Make a scatter plot of the (L1, L2) values. Then, graph $y = x^2$ in the same window. The graph of $y = x^2$ shows the relationship between any number and its square.



$[-12, 12, 1, -10, 120, 10]$

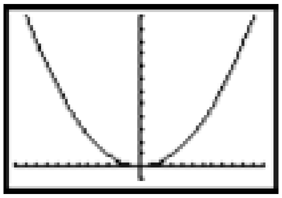
You can use the vertical line test to verify that $y = x^2$ is a function. The domain of this function is the set of all real numbers. The range is the set of real numbers greater than or equal to 0.

Steps 6–10 The graph of $y = x^2$ is a **parabola.** Except for (0, 0), all the points on the parabola are in Quadrants I and II. Looking at the graph, you can see that every output value, except 0, corresponds to two input values. For example, the output 25 corresponds to the inputs -5 and 5, and the output 6.25 corresponds to the inputs -2.5 and 2.5.

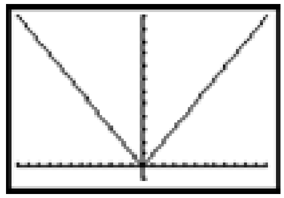
(continued)

A vertical line drawn through the origin divides the parabola into halves that are mirror images. This line (which is the y-axis) is called a **line of symmetry.** If you folded the graph along the line, the two halves of the parabola would match exactly.

Compare the parabola with the graph of $y = |x|$. Both graphs open upward, both are continuous, both have only positive y-values, and both have the y-axis as a line of symmetry. However, the parabola is curved at the bottom, whereas the absolute-value graph is pointed.



[−12, 12, 1, −10, 120, 10]



[−12, 12, 1, −1, 12, 1]

The x- and y-coordinates of each point on the parabola in the first quadrant could represent the side length and the area of a square. For example, the point (8, 64) represents a square with side length 8 and area 64.

Read the rest of the lesson in your book, which discusses how to "undo" the squaring function. Then, read the example below.

EXAMPLE Solve the equation $x^2 - 45 = 19$ symbolically.

Solution

$x^2 - 45 = 19$	Original equation.
$x^2 = 64$	Add 45 to both sides.
$\sqrt{x^2} = \sqrt{64}$	Take the square root of both sides.
$\|x\| = 8$	$\|x\|$ is the positive square root of $\sqrt{x^2}$, and 8 is the positive square root of 64.
$x = 8$ or $x = -8$	There are two solutions.

