

Relative Frequency Graphs

In this lesson you will

- create **circle graphs**
- compute **relative frequencies**
- create **relative frequency bar graphs** and **relative frequency circle graphs**

Both bar graphs and circle graphs summarize data that is grouped into categories. **Relative frequency graphs** show the percent of the total value each category represents. In this lesson you will see how to make relative frequency circle graphs and bar graphs.

Investigation: Circle Graphs and Bar Graphs

The bar graph on page 550 of your book shows the land area of each of the seven continents. You can use the graph to approximate the area of each continent and the total area.

To convert the data into a circle graph, you need to figure out the angle measure for each section of the graph. To do this, use the fact that there are 360 degrees in a circle.

For example, to find the number of degrees in the section representing Australia, solve this proportion using the data in the table at right.

$$\frac{\text{Land area of Australia}}{\text{Total land area of all continents}} = \frac{\text{Angle measure for Australia section}}{\text{Total of angle measures for all sections}}$$

$$\frac{7}{147} = \frac{x}{360}$$

Continent	Land area (millions of km ²)
Australia	7
Europe	9
Antarctica	14
South America	18
North America	24
Africa	30
Asia	45
Total	147

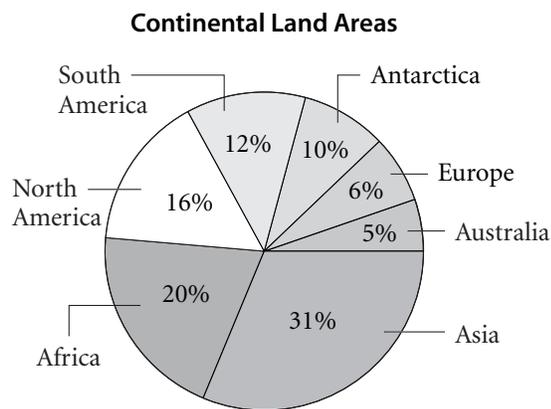
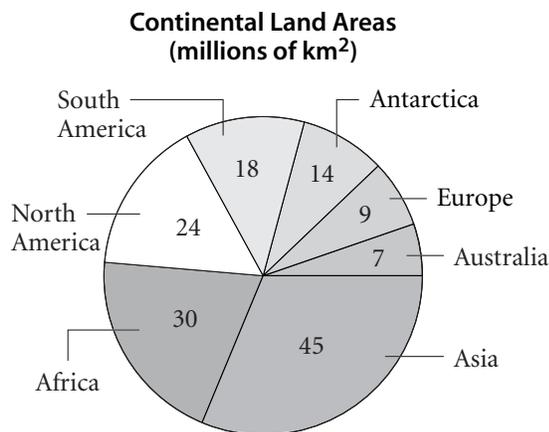
This table shows the angle measure for each section:

Continent	Angle measure
Australia	17°
Europe	22°
Antarctica	34°
South America	44°
North America	59°
Africa	73°
Asia	110°

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Lesson 10.1 • Relative Frequency Graphs (continued)

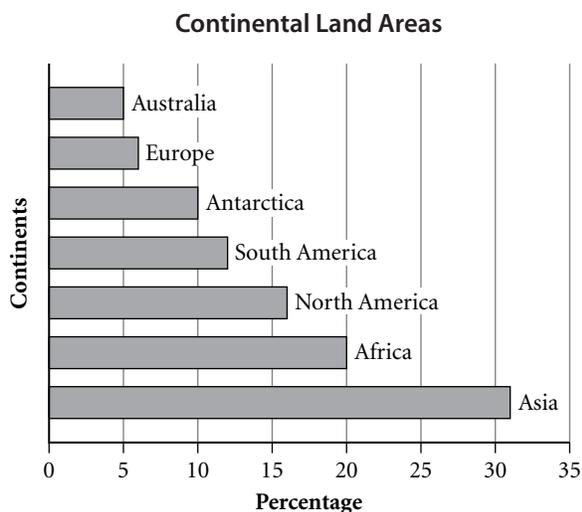
Below at left is the finished graph. To change the graph to a relative frequency circle graph, label each section with the percent of the total area the continent contains. You can calculate the percents by writing and solving proportions. For example, to find the percent of the total area Australia contains, you could solve the proportion $\frac{7}{147} = \frac{a}{100}$. Below at right is the same circle graph with the sections labeled with percents.



In both circle graphs, the relative size of each section indicates the portion of the total area that continent contains.

A relative frequency bar graph for this data shows percents of the total land area rather than the land area itself. At right is the completed relative frequency bar graph.

Notice that, like box plots, relative frequency plots do not show actual data values. For example, both of the relative frequency graphs show that Asia makes up 31% of the total land area of the continents, but neither graph indicates what the land area of Asia is.



Example A in your book shows you the steps for creating a relative frequency circle graph and bar graph for a different set of data. In the example, a calculator is used to compute the angle measures for the circle graphs and to calculate relative frequencies. Read through this example and follow along on your calculator.

Example B shows you how to use percents to find the chance that a random event will happen. Read this example carefully.

Probability Outcomes and Trials

In this lesson you will

- calculate **experimental probabilities** or **relative frequencies** for events
- calculate **theoretical probabilities** for events
- compare experimental probabilities to theoretical probabilities

A **probability** of an **event** or **outcome** is a number between 0 and 1 (or 0% and 100%) that expresses the chance the event will happen. You can find a probability by calculating the ratio of the number of ways the event can occur to the total number of ways under consideration. For example, the probability of tossing a coin and getting heads is $\frac{1}{2}$ because one of the two possible tosses is heads.

EXAMPLE

Joe works in a train station selling coffee and orange juice to commuters. Last Tuesday he sold 60 large coffees, 25 small coffees, 45 large orange juices, and 20 small orange juices. If this distribution accurately reflects the preferences of his customers, what is the probability his first customer next Tuesday will buy a large orange juice? A coffee? Tea?

► Solution

The probability that the customer will buy a large orange juice can be expressed as the ratio

$$\frac{\text{number of customers who bought a large orange juice}}{\text{total number of customers}} = \frac{45}{150} = 0.3$$

The probability that the customer will buy coffee is

$$\frac{\text{number of customers who bought coffee}}{\text{total number of customers}} = \frac{60 + 25}{150} = \frac{85}{150} \approx 0.57$$

None of the customers bought tea because it was not a choice. So the probability that the first customer will buy tea is

$$\frac{\text{number of customers who bought tea}}{\text{total number of customers}} = \frac{0}{150} = 0$$

Now, read the example in your book and the text that follows, which explains the terms **trial**, **experimental probability**, and **theoretical probability**. Think about how each of these terms applies to the example above.

Investigation: Candy Colors

This investigation involves finding the probabilities of selecting various colors from a bag of candy. By conducting an experiment, you can find experimental probabilities, or *relative frequencies*. By counting the number of candies of each color, you can find theoretical probabilities.

Steps 1–2 To conduct the experiment, you randomly choose a candy from the bag, record the color, and return the candy to the bag. You repeat this process 40 times. The total number of times each color is selected is called the color's **experimental frequency**.

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Lesson 10.2 • Probability Outcomes and Trials (continued)

From the experimental frequencies and the total number of trials (40), you can calculate the experimental probability, or relative frequency, of each color. For instance, the experimental probability of choosing a red candy will be

$$\frac{\text{number of red candies drawn}}{\text{total number of trials}}$$

Here are the data and experimental probabilities that one group found.

	Experimental Outcomes						Total
	Red	Orange	Brown	Green	Yellow	Blue	
Tally							40
Experimental frequency	3	13	8	5	10	1	40
Experimental probability (relative frequency)	$\frac{3}{40} = 7.5\%$	$\frac{13}{40} = 32.5\%$	$\frac{8}{40} = 20\%$	$\frac{5}{40} = 12.5\%$	$\frac{10}{40} = 25\%$	$\frac{1}{40} = 2.5\%$	100%

Because the table accounts for every possible color, the probabilities add to 1, or 100%.

Steps 3–6 The group who collected the data above dumped out all the candies and counted the number of each color. They then calculated the theoretical probability of drawing each color. For example,

$$P(\text{red}) = \frac{\text{number of red candies in the bag}}{\text{total number of candies in the bag}}$$

Here are their results.

	Outcomes						Total
	Red	Orange	Brown	Green	Yellow	Blue	
Number of candies	2	14	14	10	12	4	56
Theoretical probability	$\frac{2}{56} = 3.6\%$	$\frac{14}{56} = 25\%$	$\frac{14}{56} = 25\%$	$\frac{10}{56} = 17.9\%$	$\frac{12}{56} = 21.4\%$	$\frac{4}{56} = 7.1\%$	100%

In this situation, each candy is equally likely to be selected, but some colors have a higher probability of being chosen than others. The data shows that orange and brown are most likely to be chosen and that red is least likely. Notice that the theoretical probabilities, like the relative frequencies, add to 100%.

Compare the theoretical and experimental probabilities. For example, the experimental probabilities predict orange will be chosen about 1 out of 3 times. The theoretical probabilities predict orange will be chosen 1 out of 4 times. In general, the more trials you conduct, the closer the experimental probabilities will be to the theoretical probabilities.

10.3

Random Outcomes

In this lesson you will

- calculate experimental probabilities for a **random** process
- use a calculator to simulate coin tosses
- make a graph comparing experimental probabilities of tossing heads to the theoretical probability

A process is **random** if you can't predict exactly what will happen on the next trial. Read the introductory text and the example in your book. The example shows that sometimes, even when you cannot predict the exact outcome of a situation, you can use collected results to predict what will happen over the long run. Here is another example.

EXAMPLE

One afternoon, Johnna kept track of the number of cars and trucks that passed by her apartment window during a one-hour period. (She considered minivans and sport-utility vehicles to be trucks.) She counted 72 cars and 40 trucks. Use these results to predict approximately how many of the next 100 vehicles that pass by will be trucks.

► Solution

The experimental probability that a passing vehicle will be a truck is

$$\frac{\text{number of trucks}}{\text{total number of vehicles}} = \frac{40}{112} \approx 0.36 \approx 36\%$$

Johnna can calculate the *probability* that the next vehicle will be a truck, but she can't know for sure. From Johnna's perspective the event is random. Because the observed probability is 36%, Johnna can expect that 36 of the next 100 vehicles that pass by her window will be trucks.

When you toss a coin once, you cannot predict whether it will show heads or tails because the outcome is random. You do know, however, that there are two equally likely outcomes—heads or tails. Therefore, the theoretical probability of getting a head is $\frac{1}{2}$.

Investigation: Calculator Coin Toss

In this investigation, you use your calculator to simulate 100 coin tosses. You then create a scatter plot to compare the theoretical probability of getting heads to an experimental probability for 100 trials.

Steps 1–4 Read and follow along with Steps 1–4 in your book. These steps guide you to generate 100 coin tosses. When you are finished, your calculator table will show this information.

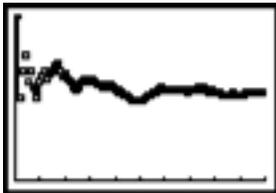
- List L1 will show the trial number.
- List L2 will show the result of each toss, with 0 representing tails and 1 representing heads. For example, the table in your book shows that the result of trial 1 was a tail and the result of trial 2 was a head.

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Lesson 10.3 • Random Outcomes (continued)

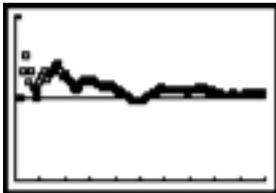
- List L₃ will show the number of heads tossed so far. In the example in your book, three heads were tossed in the first seven trials.
- List L₄ will show the experimental probability, calculated after each trial. In the example in your book, three heads were tossed in the first seven trials, so the experimental probability after seven trials is $\frac{3}{7}$ or about 0.43.

Steps 5–8 You can make a scatter plot showing the experimental probability after each trial. Use the values in list L₁ (the trial numbers) as the x -values and the values in list L₄ (the experimental probabilities) as the y -values. Because there are 100 trials, you will want to set the window to show x -values from 0 to 100. Because the highest possible probability is 1, set the maximum y -value to 1. Your plot might look something like this.



[0, 100, 10, 0, 1, 1]

Enter $\frac{1}{2}$, the theoretical probability of tossing a head, into Y₁ on the Y= screen. This graphs the line $y = \frac{1}{2}$ on the same screen as your scatter plot.



By looking at how close the points are to the line, you can compare the experimental probabilities to the theoretical probability. Notice that as the number of trials increases, the points get closer and closer to the line; that is, the experimental probabilities get closer and closer to the theoretical probability. Add data for 100 more trials to your table and make another scatter plot. You should observe that the points get even closer to the line.

The more times you toss a coin, the closer the experimental probability of getting heads will be to $\frac{1}{2}$. However, even though a pattern emerges in the long run, it will not help you predict a particular outcome. When you flip a coin, you know the theoretical probabilities for heads and tails. In some situations, you cannot calculate theoretical probabilities. In such cases, you can perform many trials and determine experimental probabilities based on your results.

10.4

Counting Techniques

In this lesson you will

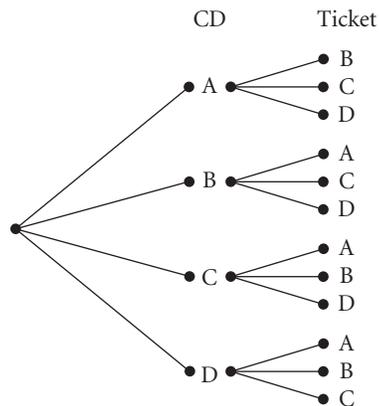
- use **tree diagrams** to help calculate probabilities
- learn the **counting principle** to determine numbers of possibilities
- learn about special types of arrangements called **permutations** and **combinations**

You've learned that the probability of an outcome is the ratio of the number of desirable outcomes to the total number of possible outcomes. Sometimes there are a lot of possible outcomes, and counting them is difficult. One way to count outcomes is to use a **tree diagram** to help organize information.

Investigation: Prizes!

Steps 1–3 If one prize is given out in a group of four people, there are four possible prize winners: A, B, C, or D.

If two prizes, a CD and a movie ticket, are given out in a group of four people, a tree diagram is useful to organize possible outcomes. The first set of branches shows who could win the CD, and for each of those branches the next set shows who could win the movie ticket.



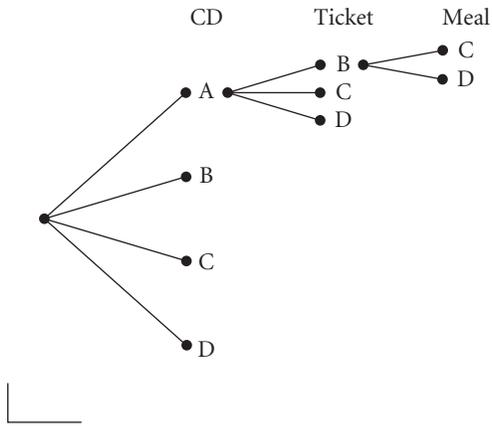
Reading across the branches, you can see that there are 12 possible sets of two winners: AB, AC, AD, BA, BC, and so on.

If three different prizes are given out to a group of four people, there are 24 possible outcomes: ABC, ABD, ACB, and so on. You don't have to draw the entire tree diagram; you need to draw just enough to see how it works.

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Lesson 10.4 • Counting Techniques (continued)

Steps 4–5 For one prize given to four people; there are four possible outcomes. For two prizes given to four people, there are $(4 \text{ choices}) \cdot (3 \text{ choices}) = 12$ possible outcomes. For three prizes given to four people, there are $(4 \text{ choices}) \cdot (3 \text{ choices}) \cdot (2 \text{ choices}) = 24$ possibilities. Following this pattern, if five students are trying out for three roles, there are $5 \cdot 4 \cdot 3 = 60$ casting arrangements.



Example A shows the same kind of situation as was presented in the investigation—an arrangement in which order is important and repeats are not allowed. (That is, the same person can't win two different prizes.) These arrangements are called **permutations**. Example C shows a different situation, called a **combination**, in which order is *not* important but repeats are still not allowed. Read the examples and text and be sure you understand when order is important.

Example B shows an arrangement that is neither a permutation nor a combination, because repeats *are* allowed. The **counting principle**, described on page 572, can be used to find numbers of arrangements in all these examples.

Multiple-Stage Experiments

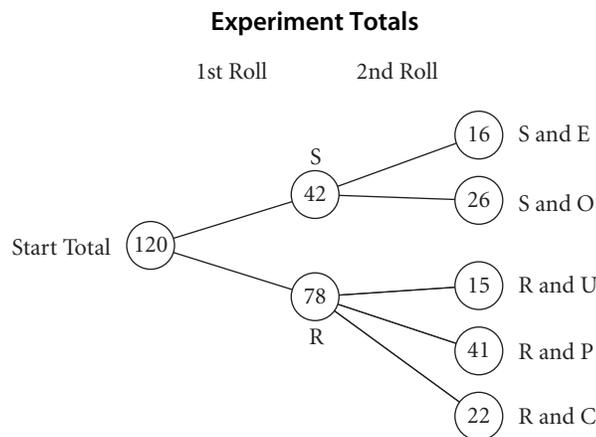
In this lesson you will

- learn the **multiplication rule** for calculating more complicated probabilities
- study **independent** and **conditional** (or **dependent**) probabilities

The investigation demonstrates how to calculate probabilities along the branches of a tree diagram.

Investigation: Pinball Pupils

Steps 1–5 Read the investigation in the book. A class of 30 students performing Steps 1–3 four times produced these results.



The sum of the numbers at the five ending points is $16 + 26 + 15 + 41 + 22 = 120$. This makes sense because 30 students did the simulation four times, so there are 120 results.

Steps 6–11 Out of 120 students, 42 went to “Square,” so $P(S) = \frac{42}{120} = 0.35$. Similarly, $P(R) = \frac{78}{120} = 0.65$. Note that $0.35 + 0.65 = 1$.

Out of 42 students at S, 16 went to E, so $P(E) = \frac{16}{42} \approx 0.38$. Similarly, $P(O) = \frac{26}{42} \approx 0.62$. Using the 78 students at R, you can calculate $P(U) = \frac{15}{78} \approx 0.19$, $P(P) = \frac{41}{78} \approx 0.56$, and $P(C) = \frac{22}{78} \approx 0.28$. (Note: These probabilities add to 0.99 rather than 1 because of rounding error.)

$P(S \text{ and } E)$ indicates the probability of a student going to S and then to E. There are 16 of these students, out of 120 students total. So $P(S \text{ and } E) = \frac{16}{120} \approx 0.13$. Calculate each of the remaining probabilities, and check that you get 0.22, 0.13, 0.34, and 0.18.

Check that $P(S) \cdot P(E) = P(S \text{ and } E)$. The same relationship holds for each of the paths.

Note that the sum of the five ending probabilities is 1. This makes sense because 100% of the students must end up in one of the final circles.

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Lesson 10.5 • Multiple-Stage Experiments (continued)

Step 12 You can create a tree diagram with theoretical probabilities rather than experimental probabilities by considering the probability of each outcome when you roll a die. For example, $P(S) = P(1 \text{ or } 4) = \frac{2}{6}$, or $\frac{1}{3}$, and $P(E) = P(2 \text{ or } 3 \text{ or } 5 \text{ or } 6) = \frac{4}{6}$, or $\frac{2}{3}$. Find the probabilities $P(U)$, $P(P)$, and $P(C)$. Then, multiply along the branches to find the probabilities of each of the final outcomes. Check that their sum is 1. Your results should be close to the experimental results found in Steps 7–9.

When considering an event with multiple stages, the probability of any given sequence of events can be found by multiplying the probability of each outcome. This is called the **multiplication rule**.

Example A shows how to use the multiplication rule to calculate the theoretical probabilities involved when flipping two coins. When you flip a coin and then flip another coin, the two events are **independent**. That is, the outcome of the first flip does not affect the outcome of the second flip—if you flip a head first, you are not any more or less likely to flip a head the second time.

Some events are **conditional** (or **dependent**), such as the event in Example B. Read Example B and notice the notation for conditional probability.

10.6

Expected Value

In this lesson you will

- learn about **expected value**

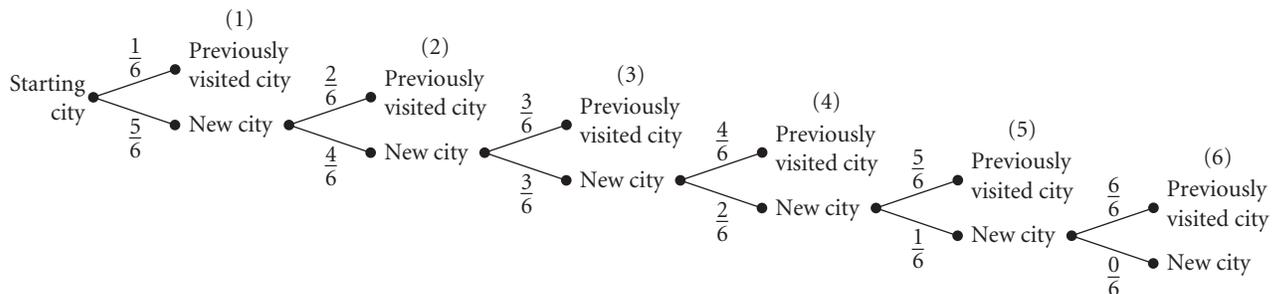
This lesson introduces **expected value**, which is a type of average. Expected value can be calculated using probabilities. Read the text before the investigation on page 584 to see an example of an expected value situation.

Investigation: Road Trip

Steps 1–5 Try Steps 1–3 a few times, and find the average of the number of cities visited on each trip. You can also run the calculator simulation CITIES several times and compare the average it produces with the average you got by rolling a die.

Step 6 You can use a tree diagram to represent this situation. On it, you can record the probability for each roll of visiting a new city or a previously visited city. There are six cities, so you won't roll the die more than six times. On the seventh roll, you're guaranteed to get a number you rolled before.

On the first roll, you determine your starting city. On the second roll, the probability is $\frac{1}{6}$ that you'll visit the city you visited before. The probability is $\frac{5}{6}$ that you'll visit a new city. If you visit the previously visited city, your trip is finished. If you do not, you roll again. This time, you've visited two cities, so the probability is $\frac{2}{6}$ that you'll visit a city you visited before. The probability is $\frac{4}{6}$ that you'll visit a new city. Here is the complete tree diagram, with the probability of each outcome. The number in parentheses indicates the number of cities visited.



Steps 7–8 To calculate the probability of visiting each number of cities, multiply the probabilities along the branches. So, $P(1) = \frac{1}{6} \approx 0.17$ (where $P(1)$ means the probability of visiting one city), $P(2) = \frac{5}{6} \cdot \frac{2}{6} = \frac{5}{18} \approx 0.28$, $P(3) = \frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} = \frac{5}{18} \approx 0.28$, $P(4) = \frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} \cdot \frac{4}{6} = \frac{5}{27} \approx 0.19$, $P(5) = \frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} \cdot \frac{2}{6} \cdot \frac{5}{6} = \frac{25}{324} \approx 0.08$, and $P(6) = \frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} \cdot \frac{2}{6} \cdot \frac{1}{6} \cdot \frac{6}{6} = \frac{5}{324} \approx 0.02$. Also, $\frac{1}{6} + \frac{5}{18} + \frac{5}{18} + \frac{5}{27} + \frac{25}{324} + \frac{5}{324} = 1$.

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Lesson 10.6 • Expected Value (continued)

The expected value (the expected number of cities you will visit) is $\frac{1}{6} \cdot 1 + \frac{5}{18} \cdot 2 + \frac{5}{18} \cdot 3 + \frac{5}{27} \cdot 4 + \frac{25}{324} \cdot 5 + \frac{5}{324} \cdot 6 = 2\frac{251}{324} \approx 2.77$. This number should be close to your results in Steps 1–5.

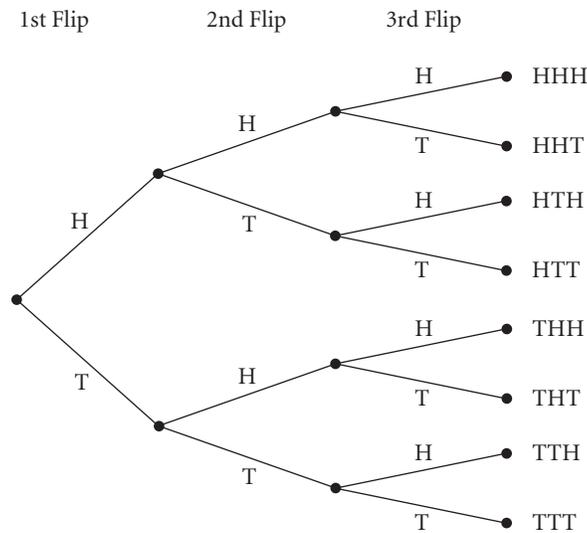
Read Examples A and B. These show how to calculate expected value in two different situations. Here is another example.

EXAMPLE

Suppose you play this game at your school's carnival: You pay \$1 to flip three pennies. If you get all heads or all tails, you win \$3. If you flip any other combination, you win nothing. What is the expected value of your earnings?

► Solution

You can make a tree diagram of the possibilities to see that there are two ways to win (all three heads or all three tails) out of eight possibilities.



So, the probability of winning \$2 (you win \$3, but pay \$1 to play) is $\frac{2}{8}$, or $\frac{1}{4}$; and the probability of losing \$1 is $\frac{6}{8}$, or $\frac{3}{4}$. You can make a table to organize this information and calculate the expected value.

Outcome	Win \$2	Lose \$1	
Probability	$\frac{1}{4}$	$\frac{3}{4}$	Sum
Product	0.50	-0.75	-0.25

So the expected value is -0.25 , meaning you expect to lose \$0.25 each time you play, on average.